The Cool-Physics Oscillator Model with Asymptotes to Predict Noise, Jitter and Size Limitations

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***Summary*—The Cool-Physics Oscillator is a significant further advance on the Leeson and subsequent Cool Oscillator Models. Its noise and jitter spectra can conveniently be expressed as log-asymptotes with the same corner frequencies for both. This log-asymptote model gives more accurate estimates of the corner-frequencies of an oscillator or jitter spectrum for sideband frequencies up to a carrier sampling-frequency or up to a newly discovered quantum physics limit. The asymptotic representation has errors only at the corner frequencies and these are less than 1.5 dB. The inclusion of f^{-1} and f^{+1} asymptotes reduces the errors to less than 0.6 dB.**

Assuming a disc or ring resonator and with a given oscillator power the resonator temperature is found to be inversely proportional to its diameter. The consequence is that a large resonator has less phase noise but is less stable but has more time jitter and worse Allan variance.

A further discovery is that around the geometric mean of the two main corner frequencies of the oscillator phase-noise spectrum multiwavelength subsidiary modes can exist with long lifetimes and these could be useful for quantum computing?

Another discovery is that the amplitude limiter in an oscillator inevitably creates some level of carrier harmonic modes which can exchange energy chaotically possibly to generate Mandelbrot fractal shapes in carrier phase-space. The energy is exchanged at very low (sideband) frequencies and also accounts for the chaotic jitter within the carrier bandwidth typically of a millihertz or so. Importantly this can be reduced if the open-loop gain of the oscillator is kept below about two.

Keywords*—*cool-oscillator; oscillator-physics; coupled-energy-modes; phase-noise; Allan-variance; asymptotic representation

I. INTRODUCTION

This paper follows on from the paper "The Cool Oscillator Coupled-Energy-Mode Model for Advanced Performance Analysis and Prediction" [A56], presented in Paris last year. An important outcome from that paper is the proposal shown here that "The Energy-Density-Asymptote (EDA) Model of Oscillator Noise" is a simple and versatile tool for understanding and optimizing the design of oscillators and oscillator systems. These asymptotes are log-log in form and as such they partition oscillator spectra into separate but partially coupled physics processes. This log-log asymptotic approach discovers and reveals several new potentially important aspects of oscillator design and performance as will be seen. This paper also follows on from previous papers listed in the author references in date order with the more recent papers being last.

Because this paper presents several very new viewpoints of oscillator understanding and design it does not contain a detailed analysis of where it attempts to progress beyond the past. Instead, only author references are included and so that what is proposed can been seen to be based on sound historical track of work spanning many years. However useful background status quo Wikipedia references are included where useful within the text of this paper.

Furthermore, it should be stressed that this is unfinished work because it opens new avenues and approaches to the future of oscillators and their central place in electronic engineering. Unfinished work rarely has useful references but it can be vital for the future development, health and survival of the topic being addressed.

The approach taken in this paper is educational. Necessary history is presented as a concise and simple summary with mathematics and symbology in agreement with the new concepts that are being presented. We therefore start with the Leeson oscillator equation somewhat simplified and reformulated. (See Wikipedia – Leeson's equation.)

We then progress on to the Cool and the Cool Physics models all presented in the very useful form of 'logarithmic-asymptotes' for phase-noise, time-jitter spectrum, and reversed (mirror-image) Allan-variance. And finish with some useful outcomes. The first is some, at least partly justified, design advice on oscillator open loop gain. And finally, some implications of the fact that it is next to impossible to create an amplitude limiting process in oscillator that does not create small but inevitable fundamental chaotic instabilities which look like Mandelbrot-fractals when the 'phase-plane' of the oscillator output is examined.

II. LEESON, COOL AND COOL-PHYICS MODEL EQUATIONS COMPARED

The original Leeson Oscillator Phase Noise Equation can be stated as:

$$L_f = \frac{\frac{1}{2}FkT}{4PQ_l^2} \times \left(\frac{f_c}{f_m}\right)^2 \qquad (1)$$

where Ql is the Q of the resonator when loaded, P is the internal power (or quadrature power) of the oscillator of which a very small fraction is tapped of as feedback to maintain the oscillation, F is the amplifier noise-factor, k is Boltzmann's constant of thermal-noise, T is the ambient noise temperature, f_c is the carrier frequency, f_m is the sideband frequency at which the phase-noise is measured and the ½ is because amplitude noise is suppressed by the 'limiter' in the oscillator setting its steady power level.

The Leeson Equation has served the oscillator community well for many decades. However, as it stands it stands it can from now on and perhaps always should be made more accurate by replacing FT by $(F\text{-}1)T = T_e$, the equivalent (input) temperature of the amplifier. This leads to the Cool-Oscillator Equation.

An oscillator is a 'saturating feedback control system' in which the Oscillator-Power P and the Resonator-Energy U are in some way limited to fixed average values. (Albeit, as addressed later, these can and do have small, arguably 'chaotic', fluctuations.) And this leads to the Cool-Oscillator Model with a phase-noise equation of:

$$L_f = \frac{\frac{1}{2}kT_e\left(f_m^2+\beta^2\right)}{PQ_l^2\ \left(f_m^2+\alpha^2\right)} \qquad (2)$$

where now $T_e = (F\text{-}1)T$, $\alpha = f_l$ now defined as the half-bandwidth of the carrier of the oscillator and $\beta = f_2$ are the two sideband frequencies where the sideband noise becomes equal to the plateau-noise kT and thereafter becomes undetectable.

A useful simplification, clarification and standardisation is to make this equation dimensionless with regard to frequency:

$$L_f = \frac{\frac{1}{2}kT_e\left(1+\beta^2/f_m^2\right)}{PQ_l^2\ \left(1+\alpha^2/f_m^2\right)} \qquad (3)$$

Equation (3) then becomes the recommended form of the Cool-Oscillator Phase-Noise Equation.

A further very useful elaboration is to turn this equation into a Laplace Transform Transfer Function (See Wikipedia- - Laplace Transform) by making the substitution $f \rightarrow s/2\pi$ where the Laplace variable $s = \sigma \pm j\omega$ and $\omega = 2\pi f$. It then becomes what is called a 'modal-equation' which is useful for defining how the oscillator power builds up when switched on and how it decays when switched off. The 'Modal' Cool-Oscillator Equation is then:

$$L_f = \frac{\frac{1}{2}kT_e\left(s^2+(2\pi\beta)^2\right)}{PQ_l^2\ \left(s^2+(2\pi\alpha)^2\right)} \qquad (4)$$

When inverted to an Inverse Laplace Transform this equation gives the switch-on and switch-off time-responses of the oscillator.

The phase noise spectrum given by any of these equations can then conveniently be plotted as log-asymptotes as shown at the top of Fig. 1.

All oscillators can be considered to be Q-Multipliers where the natural Q of the resonator is multiplied by the Q-Multiplication factor, QM, which can be defined as:

QM = *closed loop gain* $g = Q_c/Q_l = \beta/\alpha = \sqrt{(\tfrac{1}{2} FkT/PQ_l^2)}$ (3)

where the oscillator Q, Qc =fc/2α with 2α being the bandwidth of the oscillator which is typically in the few millihertz to a few microhertz region.

Treated as a control system the input is the thermal phase noise at the input of the oscillator amplifier which is amplified and filtered to give the phase-noise spectrum of the oscillator including the 'carrier' at the center of the spectrum. The carrier itself is noise extending out to the 3dB spectrum points. The as yet unanswered question is whether the carrier is a slightly chaotic mix of phase and amplitude noise? But measurements and theory indicate that there is a peak in the time jitter (and hence Allan Variance) around the 3dB carrier bandwidth frequencies.

We now find what happens when the energy density of the oscillator becomes high enough so that the quantum theory assertions that $E = hf$, Planck's relation, (see Wikipedia) and the heat spectrum-based assertion that that Planck's law (see Wikipedia) predicts a peak given by Wien's law (see Wikipedia) at a temperature T where the Total-Energy of the oscillator $E = hf_c = kT$, and f_c in this case is the carrier frequency of the oscillator.

It is now proposed that this constraint can be included in the oscillator equation to give the Cool-Physics oscillator model with the form of:

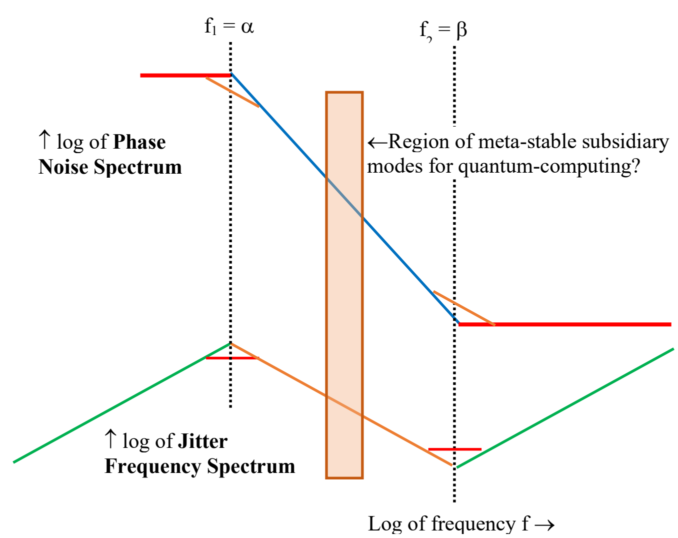
$$log_{10}\left(L_f\right) = log_{10}\left(\frac{kT_e\left(1+\frac{\beta^2}{f_m^2}\right)}{PQ_u^2\left(1+10^{\left(\frac{\alpha_h^2}{f_m^2}\right)}\right)}\right) \qquad (4)$$

Importantly in this case log10 has been taken for both sides of the original basic equation and the denominator a power of 10 factor. This ensures that the plot is scaled in dBs on both axes.

The exact value of α_h is being considered but it is thought to be k/h times an integer power of 2π, where h is Planck's constant and k is Planck's constant.

Also note that taking logs conveniently factorizes the right-hand-side of the equation so that each factor can be considered and estimated separately. For example, the 'corner-frequencies' are clearly shown in Fig. 1.

III. Asymptotic Log-Log Models of Phase Noise and Jitter



Slopes per octave constrained exactly to f^0 =+3dB green, 0 = 0dB red, f^{-1} =-3dB orange. f^{-2} = -6dB blue. f =+6dB grey. All in Upper SideBand region.

Fig. 1. The top plot is the Cool Oscillator Spectrum displayed as asymptotes. The bottom plot is the Jitter Frequency Spectrum. Note that the corner-frequencies, $f_1 = \alpha$ and $f_2 = \beta$, are the same in both plots. And also note the Metastable -Mode region which could be used for quantum computing?

Fig. 1 shows how the phase-noise and jitter spectra are directly related by having common corner frequencies of the two plots.

Of note is that the jitter frequency spectrum plot is an Allan-Variance (AVAR) plot reflected left-to-right by the equality $f = 1/\tau$ where τ is the time parameter of the AVAR.

IV. Accuracy of the Energy-Density-Asymptote (EDA) Model

Fig. 3 shows in more detail the Jitter-Density-Asymptote JDA model of Phase Noise Density compared with the Phase-Noise-Asymptote PNA model both with f^{-1} f^{0} and f^{+1} interpolation asymptotes added in red. The inclusion of these asymptotes gives a reduction of maximum estimated errors from ±1.5dB to ±0.3dB.

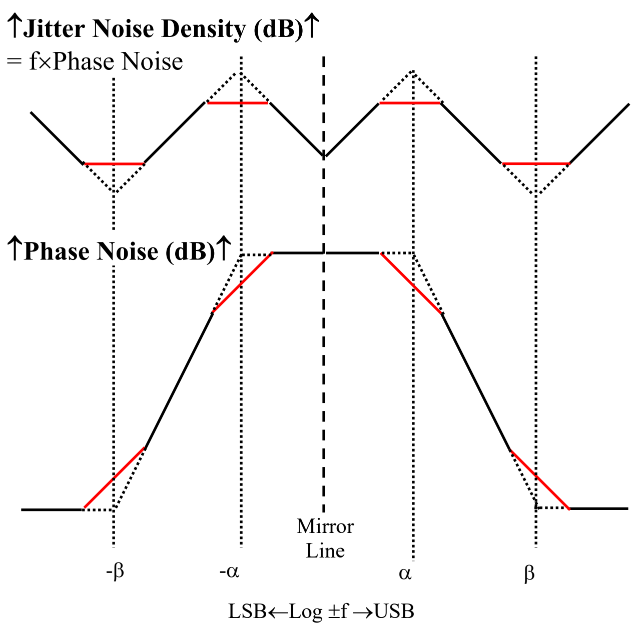


Fig. 2. JDA model at top compared with PNA model at bottom for the same generic oscillator. Also in red is shown the improved accuracy of both models when interpolation asymptote are added. Both models are for L(f) of Both upper and lower sidebands using the mirror reflection-line technique to eliminate the singularity at f=0. The corner frequencies are either 3db above or 3dB below the red interpolation asymptotes for both the PNA and JDA.

Note that the corner frequencies of the PNA and JDA models are always exactly the same for any particular oscillator.

Also note that the corner frequencies are either 3db above or below the red interpolation asymptotes.

Maximum error with the interpolation (red) asymptotes is estimated as ±0.3dB relative to Cool Oscillator or Leeson oscillator equations, or against perfect measurements.

V. Where Subsidiary Modes for Quantum Computing Can Exist in the JDA and PNA Oscillator Models

Fig1. Show where subsidiary long lasting so-called 'spurious modes' can be created and stored on any oscillator. Each such mode can store a useful amount of information as a phase and/or amplitude modulation and 'entangle' with adjacent modes in a way that is in effect a form of quantum computing. But further work is needed to find how many Qubits can be usefully generated in this way? (See Wikipedia – Qubit.)

VI. PNA Model Plot of an OCXO Phase Locked to a VCO

Fig. 4 shows the Phase-Noise-Asymptote (PNA) plot of a typical oscillator system of a VCO phase locked to an OCXO.

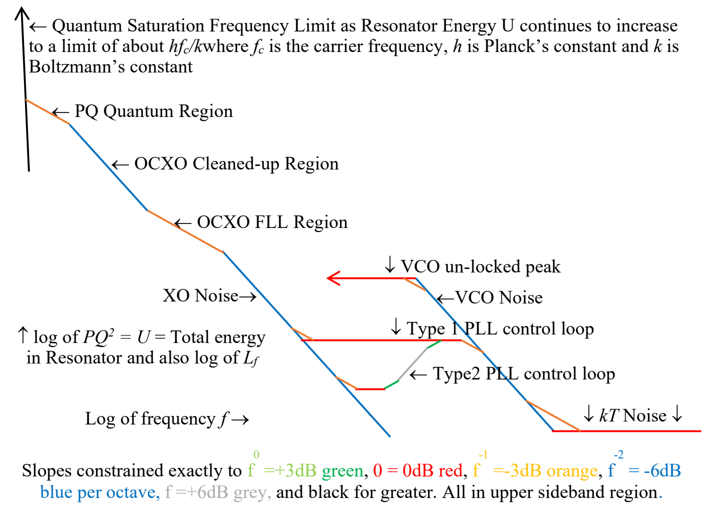


Fig. 4. Log of resonator phase-noise energy spectrum and resonator-energy PQ^2 plotted against log of frequency f for a PLL VCO locked to an OCXO for a resonator-energy PQ^2 increased to the quantum-frequency limit.

VII. PNA Model Applied to a Phase-Locked Loop and Atomic Clock Oscillator

This is shown at the top left in Fig. 4. It shows that when the energy-frequency-density (per unit of frequency) becomes high enough the Cool-Physics model equation 4 is operative and comes into effect. The out come is a 1/f (-3dB) per octave region of phase-noise leading up to a 'slightly-soft' quantum limit as shown.

This will apply when atomic clocks are used to phase lock or synchronise a system. More work is needed to quantify and validate this prediction.

VIII. JDA and PNA Models for a Cool oscillator for Different Resonator Energies and Temperatures

For a physically large oscillator-resonator a given amount of oscillator energy is spread over the large surface typically of a ring or a ring on a sphere. In both cases the peak energy density is reduced approximately as the diameter of the resonator. sphere. And this moves the oscillator further away from the quantum or energy saturation regions and the effective resonator temperature is reduced. As a consequence, phase noise is always improved for a larger resonator all else being equal.

IX. Chaotic Non Linearities from Oscillator Limiters

Fig. 5 shows that a simple diode voltage limiter in an oscillator cannot exactly limit the power of an oscillator at all times. This is for three reasons. First it only operates on one side of the resonator voltage wave form. And second it cannot operate on the resonator current waveform at exactly the same time. Note that the voltage and current in a resonator are in quadrature and are 90^0 out of phase. The third reason is the that the power waveform of a voltage or current is actually the square of a sin or cos wave and at certain times can represent negative power or negative stored energy. All this is represented in Fig. 5.

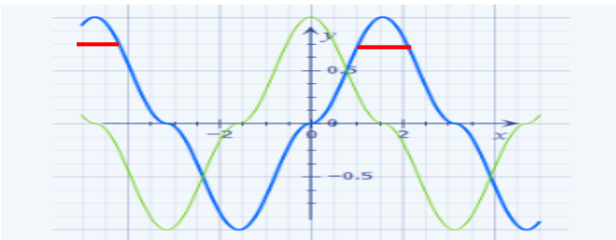


Fig. 5. We find that energy in a tuned circuit is constant but has overall has two signs, positive and negative, and has two components (green for current and blue for voltage as above) so that U = U_0 {4R(Sin(2πf)|sin(2πf)| + Cos(2πf)|Cos(2πf)|/4R }. This is fundamental and of great importance in oscillators. The red line then shows the one-sided limiting level for a single diode voltage limiter.

Equation 5 for the total energy of an oscillator as used in Fig. 6 is:

$$U = U_0 \{4R(\mathrm{Sin}(2\pi f)|\sin(2\pi f)| + \mathrm{Cos}(2\pi f)|\mathrm{Cos}(2\pi f)|/4R \} \quad (5)$$

This equation is novel and not only of great importance to the understanding and modelling of oscillator instabilities but also looks as if it should resolve some of the self-contradictions that exist elsewhere in physics, for example particularly in the theory of electromagnetic and gravity waves.

One further point of note is that on the basis of Fig, 5 the non-linearities and consequential instabilities in oscillator can be minimized by keeping the open-loop gain of the oscillator just below two. If below one the oscillator will not start and if above but close to one the oscillator will be very slow to start.

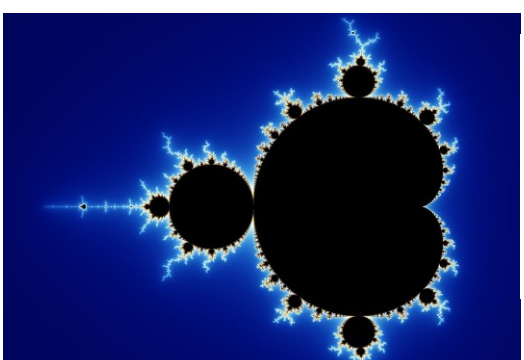


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Fig. 4. Mandelbrot fractal phase space as one possible representation of instabilities in oscillator phase space.

X. Mandelbrot Fractal Phase-Space instabilities in Voltage-Limited Oscillators?

In Fig. 5 analysis shows that possible voltage-limited phase-space trajectories could be Mandelbrot-Fractals as shown. Real component axis is horizontal and quadrature component axis is vertical. Centre large cardioid is fundamental carrier frequency. Top and bottom smaller circles are two possible phases of the second harmonic. Left-hand larger circle is the third harmonic. The smaller circles are higher harmonics that are further reduced in amplitude when the limiter is 'soft'.

The rationale for this proposal is that the 'Mandelbrot set' equation (as in Wikipedia) which generates the fractal pattern in Fig. 5 is:

$$z_{n+1} = z_n^2 + c \quad (6)$$

where c is kept less than 2 to ensure convergence.

Then after considerable mathematical manipulation in the limit this equation can be shown in the limit to represent a collection of harmonically related sine-squared and paired cos-squared waves at a carrier wave frequency and harmonics. But one where the energy in the different harmonics can switch phase in a chaotically related manner. This collection of waves in a 'phase-plane' also it appears can represent all the features of the Mandelbrot fractal of Fig. 5. More work is needed to verify this.

XI. Conclusions

This work has made further significant advances in the understanding and modelling of oscillator noise performance and instabilities. Some novel discoveries have been made which can improve oscillator understanding and lead to improved oscillator designs.

The Energy-Density-Asymptote (EDA) and Jitter Density Asymptote (JDA) Models of Oscillator Noise are simple, sufficiently accurate (±0.3dB) and versatile tools for understanding and optimizing the design of oscillators and oscillator systems.

Further work is needed on Cool-Oscillator Physics. There is much more to be discovered, particularly at optical and laser and laser oscillator frequencies.

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